DRONA

A-GSE-P-HQA

GEO-PHYSICS

Paper I

Time Allowed : Three Hours   *Maximum Marks : 200*

INSTRUCTIONS

Please read each of the following instructions carefully before attempting the questions :

There are EIGHT questions divided under TWO sections.

Candidate has to attempt SIX questions in all.

*Questions no. 1 and 5 are **compulsory**.*

Out of the remaining SIX questions, FOUR questions are to be attempted choosing TWO from each section.

The number of marks carried by a question/part is indicated against it.

All parts and sub-parts of a question are to be attempted together in the answer book.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the answer book must be clearly struck off.

Answers must be written in ENGLISH only.

Neat sketches may be drawn to illustrate answers, wherever required.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

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SECTION A

1. Attempt *all* of the following :

   (a) Discuss the magnetic elements of the Earth, with the help of a neat sketch. Give the relationships among these elements. **10**

   (b) What is fault plane solution of an earthquake ? Using the 'Beach Ball' image show the fault plane solutions for (i) Normal fault, (ii) Thrust fault, (iii) Vertical strike-slip fault, and (iv) Vertical dip-slip fault. **10**

   (c) Write down the expressions for gradient of a scalar field, T, and divergence and curl of a vector field, $\vec{A}$, in Cartesian coordinate system. **10**

   (d) What is Singular Value Decomposition (SVD) of a matrix ? How is it used to obtain the inverse of a square matrix ? **10**

2. (a) Write down the expressions of Laplace equation in Cartesian, cylindrical and spherical coordinate systems. **10**

   (b) Describe an iterative linear inversion method for quasi-linear problems and discuss the problem of trapping of the solution in local minima. **10**

   (c) Discuss the magnetic field of the Earth in terms of internal and external causes. Describe the theory, with the help of a neat sketch, that explains 90% of the observed magnetic field of the Earth. **10**

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3. (a) Describe the minimum norm solution of an underdetermined problem using Lagrange multiplier. *10*

   (b) Show that the minimum time-path between points A and B for the following ray geometry gives the same result as Snell's law : *10*
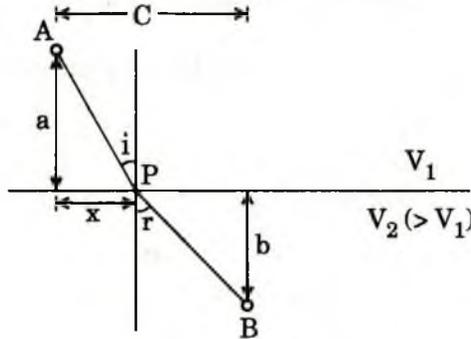
   (c) With the help of a neat sketch, describe how the time of occurrence of an earthquake and the distance travelled by the seismic wave be determined, using Wadati diagram. *10*

4. (a) What do you understand by 'Monte Carlo' and 'Directed Monte Carlo' methods ? Describe the important steps of Monte Carlo method of inversion. *10*

   (b) Discuss the mode of propagation of EM field for low (< 50 kHz) and high (≥ 50 kHz) frequencies. On the basis of it, define the quasi-static approximation and quasi-static region for low frequency EM field. *10*

   (c) Discuss the geophysical evidences of 'Seafloor spreading'. *10*

SECTION B

5. Attempt *all* of the following : $4 \times$ ●$=40$

   (a)   Expand $\cosh z - \cosh 6$ in a Taylor series in powers of $(z - 6)$. What is the radius of convergence of this power series ?   *10*

   (b)   Find the wavelength and propagation speed of radio waves of 1 MHz in copper and compare your answers with those in vacuum (given that conductivity of copper is 6×10^7 SI units, $\varepsilon_0 = 8 \cdot 85 \times 10^{-12}$ SI units, $\mu_0 = 4\pi \times 10^{-7}$ SI units; also assumed $\varepsilon \simeq \varepsilon_0, \mu \simeq \mu_0$).   *10*

   (c)   1 kg of water at 0°C is brought in contact with a heat reservoir at 100°C.

       (i)   Calculate the change in entropy of water when it attains the temperature of the reservoir.

       (ii)   Obtain the entropy change of both the reservoir and the universe.   *5+5=10*

   (d)   Draw a neat block diagram of a 3-layer model for a satellite navigation system.

         Discuss the working principle of the above navigation system.   *5+5=10*

6. (a) A periodic function with a period of 2π is given by

$$f(x) = x^2 \text{ for } -\pi \leq x \leq \pi.$$

Find out the Fourier series expansion and use your result to find the sum of the series

$$\sum_{n=1}^{\infty} 1/n^2.$$ *10*

(b) Write down Maxwell's equations for a plane electromagnetic wave propagating in free space. How do these equations differ from those in an isotropic conducting medium ?

Show that the plane electromagnetic wave $\vec{E}\,(\vec{k}, \omega)$ follows the equation

$$\nabla^2\vec{E} + \frac{\omega^2}{c^2}\vec{E} = 0.$$ *6+4=10*

(c) What is the origin of day-time attenuation of the high frequency (HF) radio waves ? Write a brief note on this specific ionospheric layer.   *2+8=10*

7. (a) Write down Simpson's rule for numerical integration of

$$\int_a^b f(x)\,dx.$$

Use the above rule for computing the value of

$$\int_1^5 x^2\,dx$$

by dividing the interval (1, 5) in four equal parts. Compare the numerical value with the exact result and give your comments.   *10*

(b)   What is atmospheric plasma ? Define electron and ion plasma frequencies in terms of the material parameters. Show that the plasma effects on electromagnetic wave propagation in the ionosphere is mainly due to the electrons.   *10*

(c)   Using the relation

$$e\,(\nu,\,T)\,d\nu = \frac{2\pi h}{c^2}\,\frac{\nu^3}{e^{\beta h\nu}-1},$$

show that the energy radiated per unit area and time in the range λ and $\lambda + d\lambda$ is

$$\frac{2\pi c^2 h}{\lambda^5}\left(e^{\beta hc/\lambda}-1\right)^{-1}d\lambda = e(\lambda,\,T)\,d\lambda.$$

Here, $\lambda = c/\nu$ and $e(\nu,\,T)$ is the black body emissivity and h is Planck's constant.

Using the variable $x = \beta hc/\lambda$, show that the value of x for which $e(\lambda,\,T)$ is maximum is given by

$$(5-x)\,e^x - 5 = 0.\qquad\qquad\textit{10}$$

8.   (a)   Define a group and show that the set of all 2×2 unitary matrices with determinant 1 is a group. Find the number of independent real parameters required to specify a group element of SU(2).   *5+5=10*

(b)   The first excited state of He-atom lies 19·82 eV above the ground state. If this excited state is 3-fold degenerate while the ground state is non-degenerate, find the relative populations of the first excited state and the ground state for He-gas in thermal equilibrium at 1000 K.   *10*

(c) Show that the refractive index of the ionospheric medium can be approximated as

$$n \cong \sqrt{1 - \frac{81\,N}{f^2}}$$

with N as electron density and f is the linear frequency in kHz.

What will happen, if the electron plasma frequency is greater than the wave frequency ?

7+3=10